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Here is perhaps the simplest example to show that in a steady-state model like David Romer's, the fact that a state has zero value does not imply that it is a fair starting point for overtime.

Suppose that if we receive a kickoff, our possession will begin either at our 10-yard line (with probability 0.25) or our 40-yard line (probability 0.75). Starting at our 10-yard line, we score a FG for certain, whereas starting from our 40 we score a TD (and kick an extra point) for certain. The situation is completely symmetric with respect to the opponents.

The state values V_{10} and V_{40} satisfy the equations

$$V_{10} = 3 + 0.25 \hat{V}_{10} + 0.75 \hat{V}_{40}$$

$$V_{40} = 7 + 0.25 \hat{V}_{10} + 0.75 \hat{V}_{40}$$

where \hat{V}_{10} and \hat{V}_{40} are the values (to us) if the *opponents* have the ball at their 10-yard line or 40-yard line, respectively. Since $\hat{V}_{10} = -V_{10}$ and $\hat{V}_{40} = -V_{40}$, the equations become

$$V_{10} = 3 - 0.25 V_{10} - 0.75 V_{40}$$

$$V_{40} = 7 - 0.25 V_{10} - 0.75 V_{40}$$

whose unique solution is $V_{10} = 0$, $V_{40} = 4$. So, the value of starting a possession at the 10-yard line is zero. Nevertheless, if we begin OT with possession at our 10, we win the game for certain.