footballcommentary.com

Suppose there is a continuum of possible defenses, ranging from exclusively focused on the run to exclusively focused on the pass. We consider only defenses on the "efficient frontier," in other words, defenses that are as effective as possible against the pass, subject to a given effectiveness against the run. We will index these defenses by the probability that a running play succeeds against it. Formally, let θ denote the defense against which a running play has probability θ of success. Let $f(\theta)$ denote the probability that a pass play succeeds against defense θ.

As explained in the article, the equilibrium defense θ^* satisfies $f(\theta^*) = \theta^*$. In other words, in equilibrium, run and pass plays must be equally likely to succeed.

The offense will use a randomized strategy in which it runs with probability p^* and passes with probability $1 - p^*$. We can determine p^* by the condition that θ^* is optimal against it: The solution to

$$\min_{\theta} \; p^* \theta + (1 - p^*) f(\theta)$$

must be $\theta = \theta^*$. This implies

$$p^* + (1 - p^*) f'(\theta^*) = 0$$

so that

$$p^* = \frac{-f'(\theta^*)}{1 - f'(\theta^*)}$$

In equilibrium, then, the defenders use defense θ^* and the offense uses a randomized strategy in which it runs with probability p^*.

The negative number $f'(\theta^*)$ represents the change in effectiveness against the pass per unit change in effectiveness against the run. If this is small in absolute value, then p^* is small, and in equilibrium the offense will usually pass. Conversely, if $f'(\theta^*)$ is large in absolute value, then p^* is large, and in equilibrium the offense will usually run.