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Let p_{TD} be the probability of success for the initial TD, if we decide to attempt it. If instead we decide to settle for the FG, let p_{FG} be the probability that it is successful. Let p_1 be the probability of a successful kicked extra point, let p_2 be the probability of a successful 2-point conversion, and let p_{OT} be the probability that we win if the game goes to OT. For cases in which our initial score is a TD, let p_{FG2} be the probability that we score a subsequent FG, and let p_{TD2} be the probability that we score a subsequent TD. Finally, let q_{TD2} be the probability that we score a subsequent TD if we treat the entire field as "four-down" territory and eschew a field goal. Note that $q_{\mathrm{TD2}} \geq p_{\mathrm{TD2}}$.

We will first find conditions under which we should attempt a 2-point conversion if we go for the initial TD and succeed. Assuming we have already scored the initial TD, our probability of winning the game if we decide to kick the extra point is

$$p_1(p_{\mathrm{FG2}}p_{\mathrm{OT}} + p_{\mathrm{TD2}}) + (1 - p_1)q_{\mathrm{TD2}}.$$

Alternatively, if after scoring the initial TD we decide to go for 2, our probability of winning the game is

$$p_2(p_{\mathrm{FG2}} + p_{\mathrm{TD2}}) + (1 - p_2)q_{\mathrm{TD2}}.$$

So, it's better to go for 2 if

$$p_1\left(\frac{p_{\mathrm{FG2}}}{p_{\mathrm{TD2}}}p_{\mathrm{OT}} + 1\right) + (1 - p_1)\frac{q_{\mathrm{TD2}}}{p_{\mathrm{TD2}}} < p_2\left(\frac{p_{\mathrm{FG2}}}{p_{\mathrm{TD2}}} + 1\right) + (1 - p_2)\frac{q_{\mathrm{TD2}}}{p_{\mathrm{TD2}}}.$$

This shows that the probabilities of a subsequent TD or FG affect the decision only through the ratios $p_{\mathrm{FG2}}/p_{\mathrm{TD2}} \equiv R$ and $q_{\mathrm{TD2}}/p_{\mathrm{TD2}}$.

Suppose that, after inserting appropriate values into this inequality, we determine that if we try for the initial TD and succeed, we will attempt a 2-point conversion. Then if we decide to try for the TD, our probability of winning the game is

$$p_{\mathrm{TD}}\left[\, p_2(p_{\mathrm{FG2}} + p_{\mathrm{TD2}}) + (1 - p_2)q_{\mathrm{TD2}} \,\right],$$

whereas if we settle for the FG our probability of winning is

$$p_{\mathrm{FG}} q_{\mathrm{TD2}} p_1 p_{\mathrm{OT}}.$$

It follows that we should try for the TD if

$$\frac{p_{\mathrm{TD}}}{p_{\mathrm{FG}}} > \frac{\frac{q_{\mathrm{TD2}}}{p_{\mathrm{TD2}}} p_1 p_{\mathrm{OT}}}{p_2 \left[\frac{p_{\mathrm{FG2}}}{p_{\mathrm{TD2}}} + 1\right] + (1 - p_2)\frac{q_{\mathrm{TD2}}}{p_{\mathrm{TD2}}}}.$$

Alternatively, suppose we have determined that if we try for the initial TD and succeed, we will kick the extra point. Then if we decide to try for the TD, our probability of winning the game is

$$p_{\mathrm{TD}} \left[\, p_1(p_{\mathrm{FG2}} p_{\mathrm{OT}} + p_{\mathrm{TD2}}) + (1 - p_1) q_{\mathrm{TD2}} \,\right],$$

while the probability that we win the game if we initially try a FG remains

$$p_{\mathrm{FG}} q_{\mathrm{TD2}} p_1 p_{\mathrm{OT}}.$$

It follows that we should try for the TD if

$$\frac{p_{\mathrm{TD}}}{p_{\mathrm{FG}}} > \frac{\frac{q_{\mathrm{TD2}}}{p_{\mathrm{TD2}}} p_1 p_{\mathrm{OT}}}{p_1 \left[\frac{p_{\mathrm{FG2}}}{p_{\mathrm{TD2}}} p_{\mathrm{OT}} + 1\right] + (1 - p_1)\frac{q_{\mathrm{TD2}}}{p_{\mathrm{TD2}}}}.$$